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A land vehicle tracking algorithm using stand-alone GPS

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Abstract

This paper describes a design of tracking algorithm for a land vehicle using stand-alone GPS. The algorithm comprised three major parts — a vehicle motion change detector, acceleration estimator and finally a Kalman filter with feedback. It contains two new techniques, acceleration estimation using a sliding mode and a vehicle motion change detection algorithm, which are developed to improve navigation accuracy. The algorithm provides more accurate position information than the conventional iterated least-squares solution, and it can easily be applied to a real-time route guidance system. Real experimental results are presented to demonstrate the advantages of the proposed algorithm. © *2000 Published by Elsevier Science Ltd.*

Keywords: Global positioning systems; Navigation; Kalman filters; Sliding mode; Detection algorithms

1. Introduction

The global positioning system (GPS) is capable of providing user position and velocity information by means of a hand-held portable receiver. Measurements of GPS include selective availability (S/A), clock bias and error from reflection, etc. The final error is about 100 m of RMS $X - Y$ coordinate. The 100 ms accuracy is not sufficient for intelligent transportation systems (ITS) and automatic vehicle location (AVL) systems. Because the amount of error can easily mean a vehicle on the wrong road in many situations, such as in dense urban areas or where a frontage road runs adjacent to a highway. These errors can result in fatal errors of ITS or AVL that might be used in operation of ambulances, police car and fire trucks, etc. To minimize such risks, it is especially important to have precise information about vehicle position and velocity. To improve the accuracy of the navigation system, there are two criteria that have to be met. The first criterion is to improve the GPS receiver itself, by selecting a satellite (Park, Kim, Lee & Jee, 1996) multi-channel filtering method (Nardi & Pachter, 1998) and differential GPS, etc. The other, with which this research is concerned, is the method of using vehicle dynamics, integrating acceleration, and then incorporating a feedback system such as that used in tracking a maneuvering target airplane (Blair, 1993). In this paper, to accurately track land vehicles using stand-alone GPS signals, modeling of a vehicle, an acceleration detector using position and dilution of precision (DOP) information, and a feedback algorithm using a sliding mode for Kalman filter are suggested and applied to real GPS measurement experiment.

2. Model for vehicle navigation

Modern optimal filter theory that was first introduced by Kalman has two distinctive features. One is its recursive form and the other is the fact that it is developed in state space instead of frequency domain. State estimators in state space need modeling of the target object, and a vehicle traveling on a road can be modeled as constant velocity (CV) model because a large portion of vehicle navigation is classified as constant velocity. This technique is similar to that used in airplane tracking algorithms (Moose, Van Landingham & McCabe, 1979). The advantages of the CV model are: simple calculation cost; very low steady-state errors; and the lack of disturbance by higher states during normal operation. Fig. 1 shows

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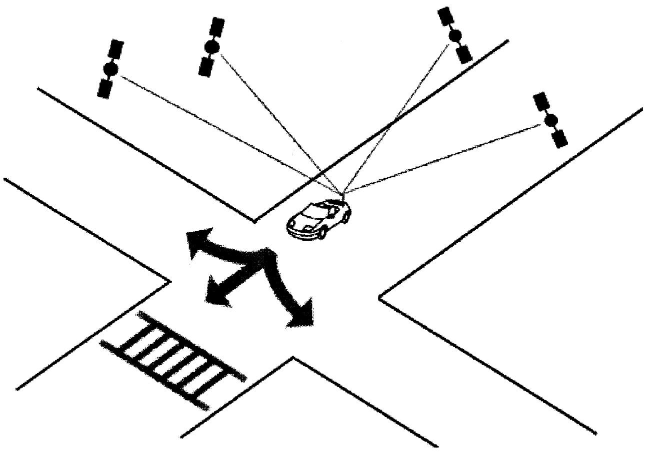

Fig. 1. A vehicle equipped with GPS.

an illustration of a vehicle equipped with GPS on the road.

A vehicle with a GPS receiver may go straight, turn right, or stop at a pedestrian crossing. Actually the CV model of a vehicle gets disturbed at the time of turning, accelerating and decelerating just as when pilots induce maneuvers in an airplane. These changes of vehicle motion act like disturbances to the filter algorithms and degrade performance. Therefore it is necessary that an efficient feedback structure is in place when the motion of vehicle changes. The CV model of navigation vehicle can be described by the following equations:

$$\mathbf{x}_{k+1} = \mathbf{F}_k \mathbf{x}_k + \mathbf{B}_k \mathbf{a}_k, \tag{1}$$

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + v_k, \tag{2}$$

$$\mathbf{x}_k = \begin{bmatrix} x_p \\ x_v \\ y_p \\ y_v \end{bmatrix}_k, \quad \mathbf{F}_k = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{H}_k^{\mathrm{T}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{B}_k = \begin{bmatrix} \frac{1}{2}\Delta t^2 & 0 \\ \Delta t & 0 \\ 0 & \frac{1}{2}\Delta t^2 \\ 0 & \Delta t \end{bmatrix} \tag{3}$$

and the acceleration vector

$$\mathbf{a}_k = \begin{bmatrix} a_x \\ a_y \end{bmatrix}.$$

Δt is the sampling time. The observation $\mathbf{z}_k$ is corrupted by the noise v_k that is assumed to be Gaussian that has a variance of R_k from various sources of selective availability (S/A), multipath, and clock errors, etc. Among the various GPS error sources, the largest portion is due to S/A that has slowly time-varying characteristics. Because the time constant of S/A is very large in comparison with the vehicle's speed, the effect of S/A is represented as a low-frequency drift that is filtered to a certain extent. The other noise sources (clock bias, drift, multipath, ghost, etc.) can be regarded as white noise. A discrete version of the Kalman filter update equation is summarized in Eqs. (4)–(8).

Time update:

$$\hat{\mathbf{x}}_{k/k-1} = \mathbf{F}_{k-1} \hat{\mathbf{x}}_{k-1/k-1}, \tag{4}$$

$$\mathbf{P}_{k/k-1} = \mathbf{F}_{k-1} \mathbf{P}_{k-1/k-1} \mathbf{F}_{k-1}^{\mathrm{T}} + \mathbf{G}_{k-1} \mathbf{Q}_{k-1} \mathbf{G}_{k-1}^{\mathrm{T}}. \tag{5}$$

Measurement update:

$$\mathbf{K}_k = \mathbf{P}_{k/k-1} \mathbf{H}_k^{\mathrm{T}} (\mathbf{H}_k \mathbf{P}_{k/k-1} \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k)^{-1}, \tag{6}$$

$$\hat{\mathbf{x}}_{k/k} = \hat{\mathbf{x}}_{k/k-1} + \mathbf{K}_k (\mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k/k-1}), \tag{7}$$

$$\mathbf{P}_{k/k} = \mathbf{P}_{k/k-1} - \mathbf{P}_{k/k-1} \mathbf{H}_k^{\mathrm{T}} (\mathbf{H}_k \mathbf{P}_{k/k-1} \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k)^{-1} \mathbf{H}_k \mathbf{P}_{k/k-1}. \tag{8}$$

3. Detection algorithm

Acceleration of a vehicle produces changes in velocity and position, which result in errors that should be corrected by feedback. Therefore, they should be detected as quickly as possible from noisy GPS measurements. If the CV model is not disturbed by acceleration, the error covariance propagation (Eqs. (5) and (8)) shows an oscillation phenomenon as in Fig. 2.

The difference between the two update processes (second term of RHP of Eq. (8)) is proportional to the inverse of the measurement error from GPS. If the vehicle is going at a constant speed, which is perfectly matched

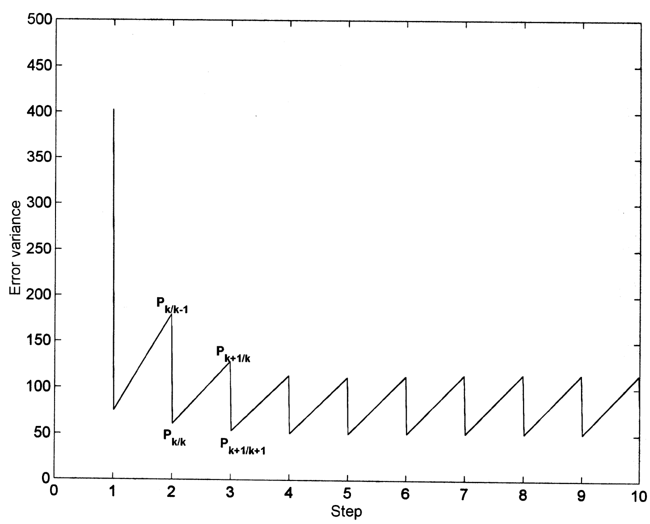

Fig. 2. Error covariance propagation of the Kalman filter.

with the CV model, then the difference of the two update processes is statistically equal to the length of the vertical line drawn in each step of Fig. 2. By using this fact, it is possible to verify whether the vehicle is accelerated (or decelerated) or not. Theorem 1 connects the measure of expectation to real measurement space.

Theorem 1. *The difference of the two covariance matrices can be described as an expectation function of innovations process and optimal gain* $\mathbf{K}$:

$$\mathbf{P}_{k/k-1} - \mathbf{P}_{k/k} = E[\mathbf{K}_k \mathbf{v}_k \mathbf{v}_k^{\mathrm{T}} \mathbf{K}_k^{\mathrm{T}}], \tag{9}$$

where innovations $\mathbf{v}_k = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k/k-1}$.

Proof. Given in Appendix A.

Theorem 1 describes about the relation between the difference of the two covariance matrices and real measurement data. The right-hand side of Eq. (9) can be calculated from innovation processes that include real measurement data at each step. Compared to the left side of the pre-determined statistical error covariance, it is possible to determine whether the velocity of the vehicle changes or not. For further development, define a process as

$$\mathbf{c}_i = \mathbf{H}_i \mathbf{K}_i \mathbf{v}_i. \tag{10}$$

The process $\mathbf{c}_i$ has zero mean and normal distribution because $\mathbf{v}_i$ is an innovation process (Kailath, 1968). Therefore, the square of process $\mathbf{c}_i$ has a chi-square distribution. For verifying chi-square distribution in real time, the Pearson's test statistic is introduced:

$$\mathbf{q}_k = \sum_{k-n}^{k} \frac{\Gamma_i(\mathbf{c}_i - E[\mathbf{c}_i])^2}{\mathrm{var}(\mathbf{c}_i)}, \tag{11}$$

where Γ_i is a weighting factor determined from DOP information from the GPS data stream and n means the size of the sample window. The variance of $\mathbf{c}_i$ is

$$E[(\mathbf{c}_i - \bar{\mathbf{c}}_i)(\mathbf{c}_i - \bar{\mathbf{c}}_i)^{\mathrm{T}}] = E[\mathbf{c}_i \mathbf{c}_i^{\mathrm{T}}] = E[\mathbf{H}_i \mathbf{K}_i \mathbf{v}_i \mathbf{v}_i^{\mathrm{T}} \mathbf{K}_i^{\mathrm{T}} \mathbf{H}_i^{\mathrm{T}}]. \tag{12}$$

By Theorem 1, the variance of $\mathbf{c}_i$ can be represented by the difference between $\mathbf{P}_{i/i-1}$ and $\mathbf{P}_{i/i}$. In the denominator of Eq. (11), if only the real measurable state is considered, the complete form of Eq. (11) using Theorem 1 is

$$\mathbf{q}_k = \sum_{k-n}^{k} \frac{\Gamma_i(\mathbf{c}_i)^2}{\mathbf{H}_i(\mathbf{P}_{i/i-1} - \mathbf{P}_{i/i})\mathbf{H}_i^{\mathrm{T}}} = \sum_{k-n}^{k} \frac{\Gamma_i(\mathbf{H}_i \mathbf{K}_i \mathbf{v}_i \mathbf{v}_i^{\mathrm{T}} \mathbf{K}_i^{\mathrm{T}} \mathbf{H}_i^{\mathrm{T}})}{\mathbf{H}_i(\mathbf{P}_{i/i-1} - \mathbf{P}_{i/i})\mathbf{H}_i^{\mathrm{T}}}. \tag{13}$$

There is not much additional computational cost to calculate the test parameter matrix $\mathbf{q}_k$, because of the matrices that are used in Eq. (13) are necessary for filter updating. If the chi-square degree of freedom term $(n-1)$ is determined appropriately by vehicle dynamics and sampling time, it is possible to use the standard table of chi-square to decide the threshold of alarm level and the probability of false alarms (PFA). For example, when the window size is 6 (degree-of-freedom is 5), 90% confidence is obtained if threshold is set about 9.236. The two diagonal components of $\mathbf{q}_k$ are compared with the threshold level at each discrete step. The weighting factor, Γ_i, is an $n \times n$ matrix that is designed so that it has a value that is proportional to the inverse of DOP, and if the DOP is below a certain level, it has a value of unity. A level of about $2.5 \sim 3$ gives good results in experiments.

4. Estimator for vehicle acceleration

At every moment of change in motion, a land vehicle has a certain acceleration that corresponds to various situations. The acceleration range and profile depends on the kind of vehicle and path on which the vehicle travels. Therefore it is necessary for the estimator to have robustness and a fast response. The sliding mode approach fulfills these requirements. In this section, an acceleration estimator using the sliding mode is designed.

The propagation algorithm of the Kalman filter has two updates. The expectation of the difference between these two updates is represented by Eqs. (14)–(16) when the land vehicle accelerates by the amount $\mathbf{a}$.

$$\begin{aligned} E[\mathbf{e}_k] &= E[\hat{\mathbf{x}}_{k/k} - \hat{\mathbf{x}}_{k/k-1}] \\ &= E[\mathbf{K}_k(\mathbf{z}_k - \mathbf{H}_k \mathbf{F}_{k-1} \hat{\mathbf{x}}_{k-1/k-1})] \qquad (14) \\ &= E[\mathbf{K}_k(\mathbf{H}_k(\mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_{k-1}\mathbf{a}_{k-1}) \\ &\quad + v_k - \mathbf{H}_k \mathbf{F}_{k-1} \hat{\mathbf{x}}_{k-1/k-1})] \qquad (15) \\ &= \mathbf{K}_k \mathbf{H}_k (\mathbf{F}_{k-1}\mathbf{x}_{k-1} - \mathbf{F}_{k-1}\hat{\mathbf{x}}_{k-1/k-1}) \\ &\quad + \mathbf{K}_k \mathbf{H}_k \mathbf{B}_{k-1} E[\mathbf{a}_{k-1}]. \qquad (16) \end{aligned}$$

Eq. (16) shows that the acceleration $\mathbf{a}$ creates a gap between the two updates (Eqs. (4) and (7)), and the weighting factor is $\mathbf{K}_k \mathbf{H}_k$. Using the result of Eq. (16), the estimation problem can be converted into an equivalent tracking problem. If the measurement update (Eq. (7)) is set as the reference trajectory, and time update (Eq. (4)) as the state that is to be driven to reference state, then the control input calculated in this tracking problem converges to the acceleration in the original estimation problem (Kim & Oh, 1998).

Chan (1991) introduced a discrete sliding mode tracking algorithm derived from a discrete Lyapunov function, which has no problem of chattering (Chan, 1991). In this section, a sliding mode acceleration estimator for a discrete Kalman filter is designed by using the conversion method that is described above. In this paper, the prediction term Δ is added using the relation of the discrete Kalman filter equation, and it makes the error state converge faster to zero on the sliding surface.

The design procedure of the discrete sliding mode is divided into two parts. The first step is to ensure that the error states remain on the discrete hyperplane of $s_k = s_{k+1}$, and the second is to design a switching action to push the states on to the surface. For notational simplicity, a one-dimensional axis will be considered in this section. Define $\mathbf{m}_k^* \equiv \hat{\mathbf{x}}_{k/k}$ and $\mathbf{m}_k \equiv \hat{\mathbf{x}}_{k/k-1}$, which have position and velocity states, respectively. Consider a system represented by

$$\mathbf{m}_{k+1} = \mathbf{F}_k \mathbf{m}_k + \mathbf{B}_k d_k, \tag{17}$$

$$\mathbf{e}_k = \mathbf{m}_k - \mathbf{m}_k^*, \tag{18}$$

$$s_k = \mathbf{G}\mathbf{e}_k, \tag{19}$$

where $\mathbf{G}$ is $1 \times n$ sliding surface matrix and d_k is the acceleration in the conversion problem, which has a relation of $\mathbf{B}_k d_k = \mathbf{K}_k \mathbf{H}_k \mathbf{B}_k a_k$.

At the first step, when the states are on the sliding surface, the steady-state condition of $s_k = s_{k+1}$ must be satisfied. This gives an equivalent law

$$\bar{d}_k = (\mathbf{G}\mathbf{B}_k)^{-1}\mathbf{G}(\mathbf{I} - \mathbf{F}_k)\mathbf{m}_k + (\mathbf{G}\mathbf{B}_k)^{-1}\mathbf{G}(\mathbf{m}_{k+1}^* - \mathbf{m}_k^*). \tag{20}$$

Consider Eq. (21) with the prediction term Δ to improve track ability and guarantee fast error convergence:

$$\begin{aligned} d_{k,\,eq} &= (1 + \Delta)\bar{d}_k \\ &= (\mathbf{G}\mathbf{B}_k)^{-1}\mathbf{G}[(\mathbf{I} - \mathbf{F}_k)\mathbf{m}_k + (\mathbf{m}_{k+1}^* - \mathbf{m}_k^*)] + \Delta\bar{d}_k, \end{aligned} \tag{21}$$

where a scalar Δ is determined from the error dynamics of the system. This equivalent law (21) must stabilize the error dynamics equation. The error equation of the estimator is

$$\begin{aligned} \mathbf{e}_{k+1} &= \mathbf{m}_{k+1} - \mathbf{m}_{k+1}^* \\ &= \mathbf{F}_k \mathbf{m}_k + \mathbf{B}_k d_{k,\,eq} - \mathbf{m}_{k+1}^* \qquad (22) \\ &= \mathbf{F}_k \mathbf{m}_k + \frac{\mathbf{B}_k \mathbf{G}}{\alpha}[(\mathbf{I} - \mathbf{F}_k)\mathbf{m}_k + \mathbf{m}_{k+1}^* - \mathbf{m}_k^*] \\ &\quad + \Delta \mathbf{B}_k \bar{d}_k - \mathbf{m}_{k+1}^* \qquad (23) \\ &= \left[\mathbf{F}_k + \frac{\mathbf{B}_k \mathbf{G}}{\alpha}(\mathbf{I} - \mathbf{F}_k)\right]\mathbf{e}_k \\ &\quad + \left[\mathbf{F}_k + \frac{\mathbf{B}_k \mathbf{G}}{\alpha}(\mathbf{I} - \mathbf{F}_k)\right]\mathbf{m}_k^* + \frac{\mathbf{B}_k \mathbf{G}}{\alpha}(\mathbf{m}_{k+1}^* - \mathbf{m}_k^*) \\ &\quad + \mathbf{B}_k \Delta \bar{d}_k - \mathbf{m}_{k+1}^* \qquad (24) \\ &= \left(\mathbf{F}_k + \frac{\mathbf{B}_k \mathbf{G}}{\alpha}(\mathbf{I} - \mathbf{F}_k)\right)\mathbf{e}_k \\ &\quad + \left(\frac{\mathbf{B}_k \mathbf{G}}{\alpha} - \mathbf{I}\right)(\mathbf{m}_{k+1}^* - \mathbf{F}_k \mathbf{m}_k^*) + \mathbf{B}_k \Delta \bar{d}_k, \qquad (25) \end{aligned}$$

where scalar $\alpha = \mathbf{G}\mathbf{B}_k$ and $(\mathbf{B}_k \mathbf{G})$ is a square matrix. If the scalar prediction term Δ is set as

$$\Delta = \mathbf{B}_k^+\left(\mathbf{I} - \frac{\mathbf{B}_k \mathbf{G}}{\alpha}\right)\frac{\mathbf{K}_k \mathbf{H}_k \mathbf{B}_k \hat{a}_{k-1}}{\bar{d}_k}, \tag{26}$$

where $\mathbf{B}_k^+$ is pseudoinverse of $\mathbf{B}_k$. Using Eq. (26), Eq. (25) gives

$$\begin{aligned} \mathbf{e}_{k+1} &= \left(\mathbf{F}_k + \frac{\mathbf{B}_k \mathbf{G}}{\alpha}(\mathbf{I} - \mathbf{F}_k)\right)\mathbf{e}_k \\ &\quad + \left(\frac{\mathbf{B}_k \mathbf{G}}{\alpha} - \mathbf{I}\right)(\mathbf{m}_{k+1}^* - \mathbf{F}_k \mathbf{m}_k^* - \mathbf{K}_k \mathbf{H}_k \mathbf{B}_k \hat{a}_{k-1}). \end{aligned} \tag{27}$$

By inserting the prediction term Δ, the second term on the right-hand side of Eq. (27) decreases significantly. As the amount of Eq. (27) decreases, the error states converge to the origin faster compared to when there is no prediction term Δ. If the external disturbance is constant, the second term on the right-hand side of Eq. (27) will be zero. Then the error states decrease exponentially.

In conventional tracking problem, the tracking reference is generated in advance, accordingly $\mathbf{m}_{k+1}^*$ is available at step k, but in our case it does not. Consider reference state $\mathbf{m}_k^*$ as

$$\mathbf{m}_k^* = \mathbf{F}_{k-1}\mathbf{m}_{k-1}^* + \mathbf{K}_k(z_k - \mathbf{H}_k \mathbf{F}_{k-1}\mathbf{m}_{k-1}^*). \tag{28}$$

Replacing $z_k = \mathbf{H}_k(\mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_{k-1}a_{k-1}) + v_k$ and using the fact that the measurement update $\mathbf{m}_k^*$ is an estimator, then expectation of Eq. (28) gives

$$\begin{aligned} \mathbf{m}_k^* &= \mathbf{F}_{k-1}\mathbf{m}_{k-1}^* + \mathbf{K}_k \mathbf{H}_k(\mathbf{F}_{k-1}\mathbf{x}_{k-1} - \mathbf{F}_{k-1}\mathbf{m}_{k-1}^*) \\ &\quad + \mathbf{K}_k \mathbf{H}_k \mathbf{B}_{k-1} E[a_{k-1}]. \end{aligned} \tag{29}$$

The second term on the right-hand side of Eq. (29) will converge to zero as the filter converges.

Therefore, the best predictor of the one-step-ahead measurement process at step k is

$$\mathbf{m}_{k+1}^* \cong \mathbf{F}_k \mathbf{m}_k^* + \mathbf{K}_k \mathbf{H}_k \mathbf{B}_k \hat{a}_{k-1}. \tag{30}$$

Using Eq. (30), the equivalent law (20) gives

$$\bar{d}_k = (\mathbf{G}\mathbf{B}_k)^{-1}\mathbf{G}(\mathbf{I} - \mathbf{F}_k)\mathbf{e}_k + (\mathbf{G}\mathbf{B}_k)^{-1}\mathbf{G}(\mathbf{K}_k \mathbf{H}_k \mathbf{B}_k \hat{a}_{k-1}). \tag{31}$$

Now, in the second step, the estimator law to transfer the state onto the sliding surface will be described. Consider Eq. (32):

$$\hat{d}_k = \bar{d}_k(1 + \Delta) + (\psi + \beta)\mathbf{e}_k - \psi_0 s_k, \tag{32}$$

where $\beta = \Delta\, \mathbf{B}_k^+ \mathbf{F}_k$, $\bar{d}_k$ is defined in Eq. (31) and Δ satisfies Eq. (26). The following theorem 2 provides a sufficient condition for the switching gain ψ to make the estimator stable.

Theorem 2. *The system will be stable, if* $0 < (\rho - \Delta) < 1$ *and the following switching conditions are fulfilled.*

$$\psi_i = \begin{cases} F_0 & \text{if} \quad \alpha e_{i,k} s_k < -\delta_i, \\ 0 & \text{if} \quad -\delta_i \leqslant \alpha e_{i,k} s_k \leqslant \delta_i \qquad \text{for} \quad i = 1, 2, \\ -F_0 & \text{if} \quad \alpha e_{i,k} s_k > \delta_i, \end{cases} \tag{33}$$

where F_0 is a positive constant, $\rho = \mathbf{GB}_k \psi_0$ and scalar $\alpha = \mathbf{GB}_k$. δ_i is determined as

$$\delta_i = \frac{F_0 \alpha^2}{2(1 - (\rho - \Delta))} |e_{i,k}| \sum_{j=1}^{n} |e_{j,k}|, \tag{34}$$

where

$$\Delta = \mathbf{B}_k^+ \left(\mathbf{I} - \frac{\mathbf{B}_k \mathbf{G}}{\alpha} \right) \frac{\mathbf{K}_k \mathbf{H}_k \mathbf{B}_k \hat{a}_{k-1}}{\bar{d}_k}. \tag{35}$$

Proof. Given in Appendix B.

While the mathematical derivation of the estimator appears to be complex course, the real implementation is not that difficult. The final form of the estimator is summarized as

$$\hat{a}_k = (\mathbf{H}_k \mathbf{K}_k)^{-1} (\bar{d}_k (1 + \Delta) + (\psi + \beta) \mathbf{e}_k - \psi_0 s_k), \tag{36}$$

where $\beta = \Delta \mathbf{B}_k^+ \mathbf{F}_k$ and Δ is Eq. (26). $(\mathbf{H}_k \mathbf{K}_k)^{-1}$ is a weighing factor from Eq. (16). The final term of Eq. (36) $\psi_0 s_k$ is inserted for the robustness and it is determined that it satisfies the condition of $0 < (\rho - \Delta) < 1$ in Theorem 2 (actually this term has a very small value). $\bar{d}_k$ is defined as in Eq. (31), and the switching gain ψ satisfies Eq. (33) in Theorem 2.

The design stages are summarized into two steps. One is to design the equivalent estimation law ($\bar{d}_k(1 + \Delta)$: Eq. (31)) and the second step is to design the switching action part ($(\psi + \beta)\mathbf{e}_k$: Theorem 2). The main design parameter in second step is to design F_0, δ_i of Theorem 2. F_0 can be determined by the maximum acceleration of the land vehicle, and δ_i can be determined by the size of the measurement noise and the filter sensitivity. More detailed design procedures about sliding mode can be found in Chan (1991) and other related references.

5. Feedback structure

If the land vehicle has a change of acceleration, the covariance of the acceleration signal is abruptly changed. Therefore, it needs a feedback algorithm to adapt the time-varying covariance.

Define $\tilde{\mathbf{x}}_{k/k} = \hat{\mathbf{x}}_{k/k} - \mathbf{x}$ and consider error covariance when there is acceleration $\mathbf{a}_k$;

$$\mathbf{P}_{k+1/k} = E[\tilde{\mathbf{x}}_{k+1/k} \tilde{\mathbf{x}}_{k+1/k}^{\mathrm{T}}]$$

$$= E[(\mathbf{F}_k \tilde{\mathbf{x}}_k - \mathbf{B}_k \mathbf{a}_k)(\mathbf{F}_k \tilde{\mathbf{x}}_k - \mathbf{B}_k \mathbf{a}_k)^{\mathrm{T}}] \tag{37}$$

$$= E[\mathbf{F}_k \tilde{\mathbf{x}}_k \tilde{\mathbf{x}}_k^{\mathrm{T}} \mathbf{F}_k^{\mathrm{T}} - \mathbf{F}_k \tilde{\mathbf{x}}_k \mathbf{a}_k^{\mathrm{T}} \mathbf{F}_k^{\mathrm{T}} - \mathbf{B}_k \mathbf{a}_k \tilde{\mathbf{x}}_k^{\mathrm{T}} \mathbf{F}_k^{\mathrm{T}} + \mathbf{B}_k \mathbf{a}_k \mathbf{a}_k^{\mathrm{T}} \mathbf{B}_k^{\mathrm{T}}]. \tag{38}$$

The expectation of the covariance $E[\mathbf{B}_k \mathbf{a}_k \mathbf{a}_k^T \mathbf{B}_k^T]$ can be reconstructed using the sliding mode estimator $\hat{\mathbf{a}}_k$ that was designed in Section 4. Using the update relations of the Kalman filter and replacing $\mathbf{a}_k$ by the acceleration estimator $\hat{\mathbf{a}}_k$ gives

$$\mathbf{P}_{k+1/k} = \mathbf{F}_k \mathbf{P}_{k/k} \mathbf{F}_k^{\mathrm{T}} + \mathbf{B}_k \hat{\mathbf{a}}_k \hat{\mathbf{a}}_k^{\mathrm{T}} \mathbf{B}_k^{\mathrm{T}}. \tag{39}$$

If the acceleration is detected, the estimated value of $\hat{\mathbf{a}}_k$ is updated to the time update of the error covariance matrix of Eq. (39). Then the error covariance of the next step is increased to the amount of the estimated acceleration exactly and Kalman gain is also increased. Therefore, the filter provides more information for the measurement process, which increases the filter's ability to track and prevent filter failure. Using the feedback scheme of the suggested filter, filter updates do not need the process noise term Q. Therefore, steady-state error is decreased significantly. The algorithm operates as an optimal filter unless acceleration is detected.

6. Experimental results

The experiments using the proposed feedback filter algorithm discussed in this paper were executed under real road conditions as a part of a car navigation system. In this experiment, the GPS receiver module of Magellan was used and it communicated to a notebook PC that had coordinate transforming and filtering software running on Windows 98. The GPS receiver outputs its value using the format of NMEA-0183 at a speed of 4800 BPS with a sampling time of 2 s. The reference map that was used in this experiment was a precise one developed by

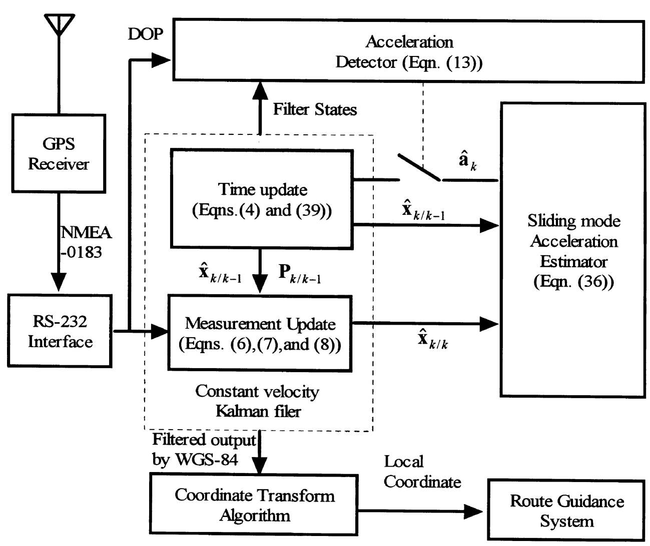

Fig. 3. Schematic diagram of the experimental system.

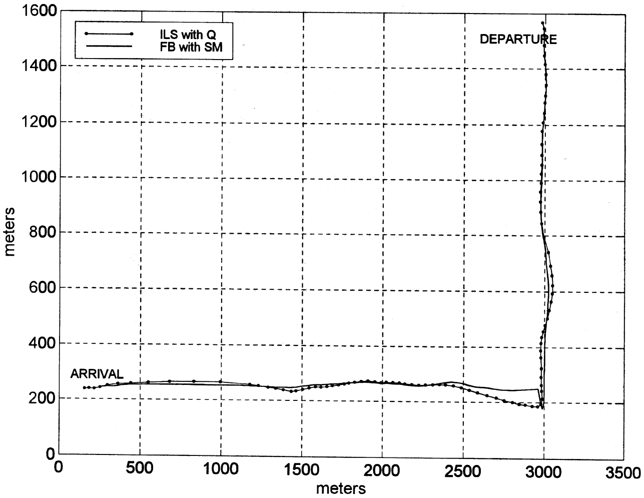

Fig. 4. Filtering result of the 'L'-shaped route.

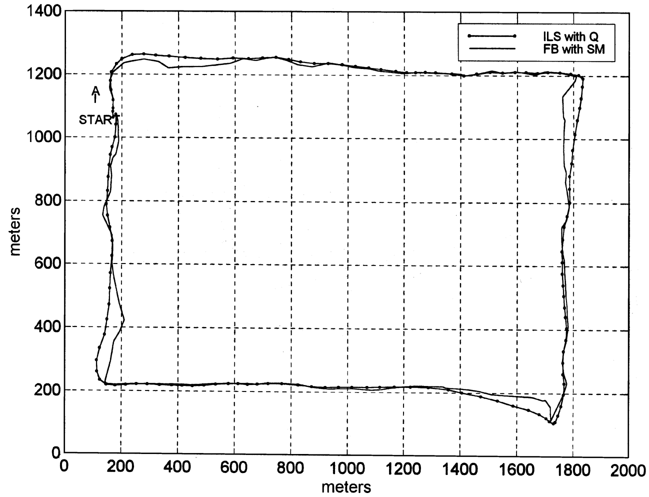

Fig. 6. Filtering result of the rectangular route.

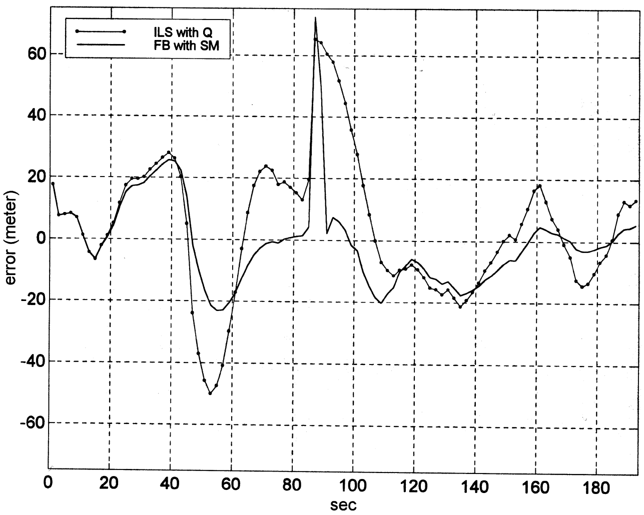

Fig. 5. Error variation of the 'L'-shaped route.

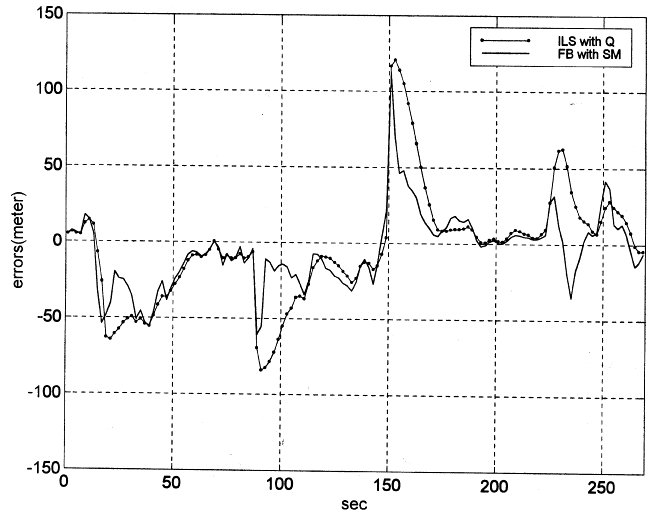

Fig. 7. Error variation of the rectangular route.

the National Geographic Department, which uses Bessel coordinates. Fig. 3 shows the proposed filter feedback structure. The numbers of equations that are used at each block are indicated. For a comparison of the proposed filter, a conventional iterative least-squares (ILS) filter algorithm that has process noise Q was also executed.

The route that is parallel or orthogonal to the latitude line was chosen for the experiment. The following simple 'L' shape route test explicitly shows the difference of the two algorithms. Fig. 4 shows an airplane view of the two filtered signals and Fig. 5 illustrates the error of the two algorithms. After 85 s, the test car that was going at a speed of 45 km/h turned right and continued going straight. In Fig. 4, the feedback filter shows a very fast recovery when acceleration was detected, and in the steady state, it rejects the low-frequency variation effectively. In the ILS algorithm, if process noise Q is increased, the filter response will be faster, but the larger process noise Q perturbs states even when there is no acceleration. Accordingly, a larger Q will increase the steady-state error, which can be seen between 140 and 190 s. in Fig. 5.

Fig. 6 shows the result of the rectangular route, and Fig. 7 illustrates the errors of two filters. At the four corners, similar features of feedback filter are noticed. The proposed filter shows fast recovery and convergence to real values. Fig. 8 shows the acceleration detecting parameter $\mathbf{q}_k$ and the feedback gain $\mathbf{K}_k$ of the position of each axis. In the experiment, the threshold of the detecting algorithm was set to the point in order for the confidence to be about 90%.

Experiments were executed on various routes that were about 5 km long at a speed of 45 km/h. Compared to the conventional ILS filter, an average improvement of 40, 16 and 27% was obtained when the route was 'L' shaped, step shaped and rectangular, respectively. There

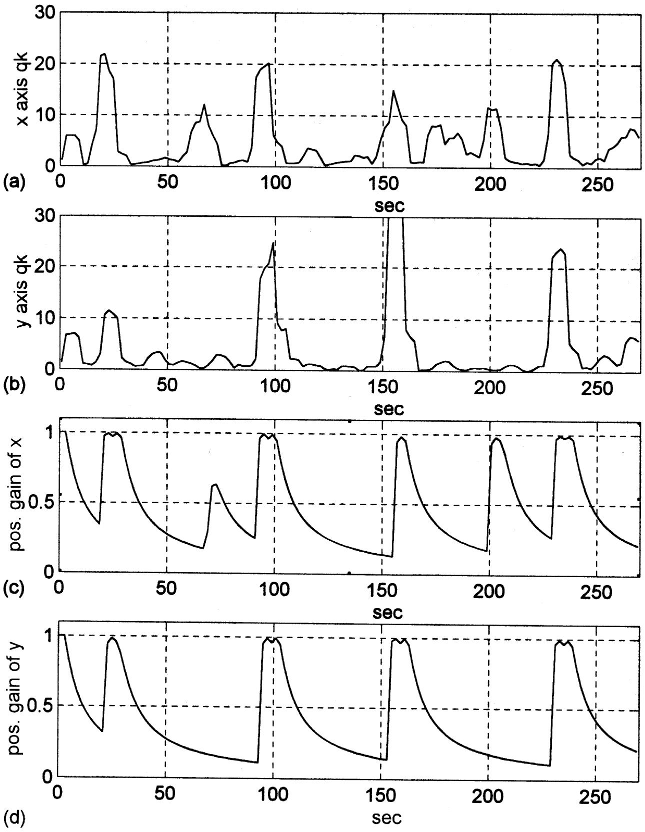

Fig. 8. Detecting parameter q_k and feedback filter gain: (a) x-axis q_k; (b) y-axis q_k; (c) position gain of x; (d) position gain of y.

is a tendency for the efficiency to decrease as the vehicle route becomes more complex.

It is known that it is possible to model the S/A as a second-order Gauss–Markov process and it has slow time varying characteristics. The time constant of S/A is very large in comparison with the vehicle's speed, and the effect of S/A can be regarded as a low-frequency disturbance, and the other noise sources (clock bias, drift, multipath, ghost, etc.) can be regarded as white noise.

7. Conclusion

A feedback filter using a sliding mode for land vehicle tracking is proposed and applied to real GPS data. For developing precise filtering output, a CV model is used. An acceleration detection algorithm is designed using position measurement and DOP information from a GPS receiver. The proposed sliding mode acceleration estimator has good characteristics to apply to various types of fast vehicles because it is robust against unmodeled acceleration. The added prediction term of the sliding mode makes the estimator more efficient, because the sampling time of the GPS measurement is relatively long compared with other control applications. The results show that the CV model and the suggested feedback scheme are effective.

Appendix A. Proof of Theorem 1

Define

$$\tilde{\mathbf{x}}_{k/k} \equiv \hat{\mathbf{x}}_{k/k} - \mathbf{x}, \tag{A.1}$$

$$\tilde{\mathbf{x}}_{k/k-1} \equiv \hat{\mathbf{x}}_{k/k-1} - \mathbf{x} \tag{A.2}$$

and the innovations process as

$$\mathbf{v}_i = \mathbf{z}_i - \mathbf{H}_i\hat{\mathbf{x}}_{i/i-1}. \tag{A.3}$$

$$E[(\hat{\mathbf{x}}_{k/k-1} - \hat{\mathbf{x}}_{k/k})(\hat{\mathbf{x}}_{k/k-1} - \hat{\mathbf{x}}_{k/k})^{\mathrm{T}}]$$
$$= E[(\tilde{\mathbf{x}}_{k/k-1} - \tilde{\mathbf{x}}_{k/k})(\tilde{\mathbf{x}}_{k/k-1} - \tilde{\mathbf{x}}_{k/k})^{\mathrm{T}}] \tag{A.4}$$
$$= E[\tilde{\mathbf{x}}_{k/k-1}\tilde{\mathbf{x}}^{\mathrm{T}}_{k/k-1} - \tilde{\mathbf{x}}_{k/k}\tilde{\mathbf{x}}^{\mathrm{T}}_{k/k-1} - \tilde{\mathbf{x}}_{k/k-1}\tilde{\mathbf{x}}^{\mathrm{T}}_{k/k} + \tilde{\mathbf{x}}_{k/k}\tilde{\mathbf{x}}^{\mathrm{T}}_{k/k}]. \tag{A.5}$$

Since

$$\tilde{\mathbf{x}}_{k/k} = \hat{\mathbf{x}}_{k/k} - \mathbf{x} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\tilde{\mathbf{x}}_{k/k-1} + \mathbf{K}_k v_k, \tag{A.6}$$

$$E[\tilde{\mathbf{x}}_{k/k-1}\tilde{\mathbf{x}}^{\mathrm{T}}_{k/k-1} - \tilde{\mathbf{x}}_{k/k}\tilde{\mathbf{x}}^{\mathrm{T}}_{k/k-1} - \tilde{\mathbf{x}}_{k/k-1}\tilde{\mathbf{x}}^{\mathrm{T}}_{k/k} + \tilde{\mathbf{x}}_{k/k}\tilde{\mathbf{x}}^{\mathrm{T}}_{k/k}] \tag{A.7}$$

$$= \mathbf{P}_{k/k-1} + \mathbf{P}_{k/k} - E[(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\tilde{\mathbf{x}}_{k/k-1}\tilde{\mathbf{x}}^{\mathrm{T}}_{k/k-1} + \mathbf{K}_k v_k\tilde{\mathbf{x}}^{\mathrm{T}}_{k/k-1} + \tilde{\mathbf{x}}_{k/k-1}\tilde{\mathbf{x}}^{\mathrm{T}}_{k/k-1}(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)^{\mathrm{T}} + \tilde{\mathbf{x}}_{k/k-1}v_k\mathbf{K}^{\mathrm{T}}_k] \tag{A.8}$$

$$= \mathbf{P}_{k/k-1} + \mathbf{P}_{k/k} - 2\mathbf{P}_{k/k} = \mathbf{P}_{k/k-1} - \mathbf{P}_{k/k}. \tag{A.9}$$

Eq. (A.4) can be described in another form as

$$E[(\hat{\mathbf{x}}_{k/k-1} - \hat{\mathbf{x}}_{k/k})(\hat{\mathbf{x}}_{k/k-1} - \hat{\mathbf{x}}_{k/k})^{\mathrm{T}}]$$
$$= E[\mathbf{K}_k(\mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_{k/k-1})(\mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_{k/k-1})^{\mathrm{T}}\mathbf{K}^{\mathrm{T}}_k] \tag{A.10}$$

where

$$\hat{\mathbf{x}}_{k/k-1} - \hat{\mathbf{x}}_{k/k} = -\mathbf{K}_k(\mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_{k/k-1}) = -\mathbf{K}_k\mathbf{v}_k \tag{A.11}$$

$$= E[\mathbf{K}_k\mathbf{v}_k\mathbf{v}^{\mathrm{T}}_k\mathbf{K}^{\mathrm{T}}_k]. \tag{A.12}$$

Appendix B. Proof of Theorem 2

The gain ψ and ψ_0 will be determined so that the estimation law of Eq. (32) satisfies the stable condition in a discrete Lyapunov function. The final term of Eq. (32) is for improving robustness (Chan, 1991). In order to ensure the stability of the proposed control law, the discrete Lyapunov function is defined as $V_k = s_k^2$. For the stability of the system

$$V_{k+1} - V_k = s^2_{k+1} - s^2_k$$
$$= 2s_k(s_{k+1} - s_k) + (s_{k+1} - s_k)^2 < 0 \tag{B.1}$$

must be satisfied. Consider Eq. (B.2),

$$s_{k+1} = \mathbf{G}(\mathbf{m}_{k+1} - \mathbf{m}^*_{k+1})$$

$$= \mathbf{G}(\mathbf{F}_k\mathbf{m}_k + \mathbf{B}_k\hat{d}_k - \mathbf{m}^*_{k+1}) \quad \text{(B.2)}$$

$$= \mathbf{G}\{\mathbf{F}_k\mathbf{m}_k + \alpha^{-1}\mathbf{B}_k\mathbf{G}[(\mathbf{I} - \mathbf{F}_k)\mathbf{e}_k + \mathbf{K}_k\mathbf{H}_k\mathbf{B}_k\hat{a}_{k-1}] \times (1+\Delta) - \mathbf{m}^*_{k+1}\} + \alpha(\psi + \beta)\mathbf{e}_k - \rho s_k, \quad \text{(B.3)}$$

where $\alpha = \mathbf{G}\mathbf{B}_k$, $\beta = \Delta\mathbf{B}_k^+\mathbf{F}_k$ and $\rho = \mathbf{G}\mathbf{B}_k\psi_0$.

Using the relation of $\mathbf{m}^*_{k+1} = \mathbf{F}_k\mathbf{m}^*_k + \mathbf{K}_k\mathbf{H}_k\mathbf{B}_k a_k$, Eq. (B.3) is

$$s_{k+1} = \mathbf{G}(1+\Delta)\mathbf{e}_k + \mathbf{G}\mathbf{K}_k\mathbf{H}_k\mathbf{B}_k((1+\Delta)\hat{a}_{k-1} - a_k) + \alpha\psi\mathbf{e}_k - \rho s_k. \quad \text{(B.4)}$$

As a_k is an external unknown acceleration that is not available at step k, it is assumed that

$$((1+\Delta)\hat{a}_{k-1} - a_k) = 0.$$

Thus by Eq. (19),

$$s_{k+1} = \mathbf{G}(1+\Delta)\mathbf{e}_k + \alpha\psi\mathbf{e}_k - \rho s_k \quad \text{(B.5)}$$

$$= \alpha\psi\mathbf{e}_k - [\rho - \Delta - 1]s_k. \quad \text{(B.6)}$$

Hence Eq. (B.1) is

$$2s_k(s_{k+1} - s_k) + (s_{k+1} - s_k)^2$$

$$= 2s_k(\mathbf{G}\Delta\mathbf{e}_k + \alpha\psi\mathbf{e}_k - \rho s_k) + (\mathbf{G}\Delta\mathbf{e}_k + \alpha\psi\mathbf{e}_k - \rho s_k)^2 \quad \text{(B.7)}$$

$$= (\alpha\psi\mathbf{e}_k)^2 + \alpha\psi s_k\mathbf{e}_k(2 - 2(\rho - \Delta)) + ((\rho - \Delta)^2 - 2(\rho - \Delta))s_k^2. \quad \text{(B.8)}$$

To make the system stable, Eq. (B.8) must have a negative value. If ψ_i is determined as (33),

$$s_{k+1}^2 - s_k^2(1 - (\rho - \Delta))^2 < \left(\sum_{i=1}^{n} \alpha\psi_i e_{i,k}\right)^2 - 2(1 - (\rho - \Delta))\sum_{i=1}^{n} F_0\delta_i. \quad \text{(B.9)}$$

By using Eq. (34),

$$s_{k+1}^2 - s_k^2(1 - (\rho - \Delta))^2 = V_{k+1} - (1 - (\rho - \Delta))^2 V_k \leqslant 0, \quad \text{(B.10)}$$

$$V_{k+1} \leqslant (1 - (\rho - \Delta))^2 V_k. \quad \text{(B.11)}$$

Since

$$(1 - (\rho - \Delta)) < e^{-(\rho - \Delta)}. \quad \text{(B.12)}$$

Eq. (B.11) is bounded as Eq. (B.13),

$$V_{k+1} \leqslant e^{-2(\rho - \Delta)} V_k. \quad \text{(B.13)}$$

Therefore, if $0 < (\rho - \Delta) < 1$ the discrete Lyapunov function has a negative semi-definite derivative and the system is stable. □

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